10. Ratio, Proportion and Rates of Change

Assessing car finance options

Mina has decided to buy her first car. After doing some online research, she is planning to go to a local car dealership this weekend to look at a few options. At the dealership they sell new cars and second hand cars and can offer financing options to buyers. Before her visit she wants to better understand what type of loan would be best and whether to look at new or used cars.

Task one: Lines of credit

Mina's bank offer her a loan of up to £7,000 with a 3.6% APR (interest is charged **monthly** on the account balance) for four years.

Annual Percentage Rate (APR) is the cost you pay each year to borrow money, including fees, expressed as a percentage.

1. If Mina takes out the full £7,000 loan and did not make any repayments, how much would she owe at the end of the four years?

£7,000 x 1.036^4 = £8,063.75

2. If Mina makes monthly repayments of £200 on the first of each month, what will the balance of her loan be after four months?

3.6% / 12 = 0.3% monthly interest
M1: (£7,000 - £200) x 1.003 = £6,820.40
M2: (£6,820.40 - £200) x 1.003 = £6,640.26
M3: (£6,640.26 - £200) x 1.003 = £6,459.58
M4: (£6,459.58 - £200) x 1.003 = £6,278.36

Mina's bank also offers her a credit card with a limit of £3,000. As part of a special promotion she is offered six months interest-free followed by an interest rate of 20% APR.

3. If she failed to make repayments on the credit card, how much would she owe after three years?

£3,000 x $1.2^{2.5}$ = £4,732.32
(Note: the power is 2.5 because the account offers six months interest-free)

4. Assuming monthly repayments of £120, how much interest will Mina be charged in the first 9 months? Mina makes the repayment before interest is charged at the end of the month.

20% APR = 1.67% per month
First 6 months: £3,000 - (£120 x 6) = £2,280 First 6 months are interest free
Month 7: £2,280 - £120 = £2,160 x 1.67% = £36.07
Month 8: (£2,160 + £36.07) - £120 = £2,196.07 x 1.67% = £34.67
Month 9: (£2,196.07 + £34.67) - £120 = £1,990.74 x 1.67% = £33.25

Total interest charged: £103.99

Mina's local car dealership offers £250 cash back if you choose their car financing plan. If she took out a £7,000 loan with them they would charge 6.2% APR for four years.

5. If Mina made no repayments on this loan how many months of interest would the £250 cash back cover?

7,000 x (6.2% / 12) = £36.17
£250 / £36.17 = 6.9 months
The cash back would fully cover the first six months of interest and most of month seven.

6. If Mina made monthly repayments of £150 before she was charged interest each month then in month eight made an additional one off repayment of £500 how much would be outstanding on the loan after one year?

M1: (£7,000 - £150) x (1 + 0.52%) = £6,885.62
M2: (£6,885.62 - £150) x (1 + 0.52%) = £6,770.65
M3: (£6,770.65 - £150) x (1 + 0.52%) = £6,655.07
M4: (£6,655.07 - £150) x (1 + 0.52%) = £6,538.90
M5: (£6,538.90 - £150) x (1 + 0.52%) = £6,422.12
M6: (£6,422.12 - £150) x (1 + 0.52%) = £6,304.74
M7: (£6,304.74 - £150) x (1 + 0.52%) = £6,186.74
M8: (£6,186.74 - £650) x (1 + 0.52%) = £5,565.53
M9: (£5,565.53 - £150) x (1 + 0.52%) = £5,443.69
M10: (£5,443.69 - £150) x (1 + 0.52%) = £5,321.22
M11: (£5,321.22 - £150) x (1 + 0.52%) = £5,198.11
M12: (£5,198.11 - £150) x (1 + 0.52%) = £5,074.36

Task two: Depreciation

Depreciation is when the value of something decreases over time.

1. If a new car depreciates by 25% in the first year and 15% per year after that, how much would you expect a new £15,000 car to be worth when it is three years old?

£15,000 x 0.75 x 0.85 x 0.85 = £8,128.13

2. [Higher] Mina finds a car model she likes and finds two second hand options in her local area. Both cars are in similar condition and have got relatively few miles on them. The one-year-old car costs £9,000 and the three-year-old model costs £6,969.60. By what percent is the car depreciating by each year?

$9000 \times (1-d)^2 = 6969.60$
$6969.60/9000 = (1-d)^2$
$6969.60/9000 = (1-d)^2$
$\sqrt{0.7744} = 1-d$
$\sqrt{0.7744} = 0.88$
Depreciation = 12%

3. [Higher] A second hand car purchased for £3,000 depreciates at 9% per year. Express this depreciation as a formula and calculate by how much the car has depreciated after three years.

$V = P \times (1-d)^n$
Where: V = value of the car; P = price of the car ; d = depreciation rate; and n = number of years.

$£3,000 \times (1-0.09)^3 = £2,260.71$
£3,000 - £2,260.71 = £739.29 total depreciation

A £4,000 car depreciates at 12% per year. 18 months after buying the car a tree falls on the car causing major damage.

4. How much was the car worth before the tree fell on it?

£4,000 x $(1\text{-}0.12)^{1.5}$ = £3,302.05

5. [Higher] A local garage tells offers to either fix the car for £1,500 or purchase it from you for £2,000. Which offer would you take and why?

The value of the car before the damage was £3,302.05. If it costs £1,500 to fix then the car is effectively only worth £1,802.05 now, so I would accept the offer of £2,000.

Task three: Comparing cars

Mina narrows down her three favourite cars:

	Car 1	Car 2	Car 3
Price	£7,000	£5,900	£5,200
Annual depreciation	25%	17%	11%

1. How much will each car be worth in five years?

Car 1: £7,000 x 0.75^5 = £1,661.13
Car 2: £5,900 x 0.83^5 = £2,324.03
Car 2: £5,200 x 0.89^5 = £2,903.71

2. [Higher] Mina buys Car 1 using the £7,000 bank loan offered by her bank (see Task One), she has been repaying £150 per month on the loan.

   After six months she changes jobs and can now walk to work so she decides to sell her car. If she puts all the car proceeds (the money she made from selling the car) towards the loan will she have paid it off in full? If not, what would be left to pay on the loan?

Value of the car after 6 months:
25% annual depreciation = 2.08% monthly
£7,000 x 0.9792^6 = £6,170.59

3.6% / 12 = 0.3% monthly interest

M1: (£7,000 - £150) x 1.003 = £6,870.55
M2: (£6,870.55 - £150) x 1.003 = £6,740.71
M3: (£6,740.71 - £150) x 1.003 = £6,610.48
M4: (£6,610.48 - £150) x 1.003 = £6,479.87
M5: (£6,479.87 - £150) x 1.003 = £6,348.85
M6: (£6,348.85 - £150) x 1.003 = £6,217.45

Loan - car sale proceeds = £6,170.59 - £6,217.45 = - £46.86